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The following is quoted from Dr. Burnet's article:
"Virulence is an inheritable character—from the point of view of medicine it is the most important character of all, and as such worthy of close genetic study. I am rather sorry that, for understandable reasons, very little refined genetic study has yet been made of the phenomena of bacterial virulence. It might be a happy thought if someone switched from *E. coli* B and K-12 to pathogenic strains of *S. pullorum* or *gallinarum* which can readily be tested for pathogenicity. . . ."

ON FINITE GROUPS OF EVEN ORDER WHOSE 2-SYLOW GROUP IS A QUATERNION GROUP

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We shall sketch a proof of the following theorem:[1]

THEOREM 1. *Let G be a group of finite even order. If the 2-Sylow group P of G is a quaternion group (ordinary or generalized), then G is not simple.*

Proof: If P has order 2^n, it can be generated by two elements α and β with

$$\alpha^{2^{n-1}} = 1, \qquad \beta^2 = \alpha^{2^{n-2}}, \qquad \beta^{-1}\alpha\beta = \alpha^{-1}.$$

The element $\mu = \alpha^{2^{n-2}}$ is the only element of order 2 in P and hence all elements of order 2 in G are conjugate to μ in G. Let $\chi_1 = 1, \chi_2, \ldots, \chi_k$ denote the irreducible characters of G and set $x_i = \chi_i(1)$. If σ is an element of G of even order, it can be shown without difficulty[2] that

$$\sum_{i=1}^{k} \chi_i(\mu)^2\chi_i(\sigma)/x_i = 0. \tag{1}$$

Let $\pi \neq 1$ be a fixed element of P and let ρ be an element of odd order of the centralizer $C(\pi)$ of π. Applying the theory of modular character for the prime $p = 2$, we have

$$\chi_i(\pi\rho) = \sum_j d_{ij}^{\pi}\varphi_j^{\pi}(\rho) \tag{2}$$

where the d_{ij}^{π} are the decomposition numbers of G for $p = 2$ and where $\varphi_1^{\pi} = 1$, φ_2^{π}, φ_3^{π}, ... are the irreducible modular characters of $C(\pi)$ for $p = 2$. On substituting (2) in (1) for $\sigma = \pi\rho$ and using the linear independence of the φ_j^{π}, we obtain

$$\sum_i \chi_i(\mu)^2\, d_{ij}^{\pi}/x_i = 0,\ (j = 1,2,\ldots). \tag{3}$$

Assume first that P is a generalized quaternion group, $n > 3$. We let π range over the elements of $\{\alpha\}$ of orders 2^{n-1} and 2^{n-2}. It can be seen that there exists a linear combination of the corresponding equations (3) with $j = 1$ of the form

$$\sum_i \chi_i(\mu)^2 t_i/x_i = 0 \tag{4}$$

such that the t_i are rational integers, $t_i = 1$, and

$$\sum_i t_i^2 = 3. \tag{5}$$

It follows from the properties of the decomposition[3] numbers that

$$(a)\ \sum_i t_i x_i = 0;\quad (b)\ \sum_i t_i\chi_i(\mu) = 0. \tag{6}$$

Because of (5), only three of the t_i are different from 0. On combining (4) and (6), we can deduce $\chi_i(\mu) = \chi_i(1)$ for these three values of i. Hence μ belongs to the kernels of the corresponding representations and G cannot be simple.

If P is the ordinary quaternion group, the argument is more complicated. We may assume that G does not have a normal subgroup of index 2. By the *principal block* of characters of a group, we mean the block which contains the principal character $\chi_1 = 1$ (for the given prime p, here $p = 2$). We take $\pi = \alpha$ and $\pi = \mu$ choosing for φ_j^{π} the irreducible modular characters of the principal block of $C(\pi)$. For $\pi = \alpha$, we have only $\varphi_1^{\mu} = 1$ while for $\pi = \mu$ we have three characters φ_j^{μ}. This gives us four equations (3). In addition, we use (2) and the properties of the decomposition numbers, in particular, the orthogonality relations.[3] If the characters χ_i are taken in a suitable order, it can be shown that there exist signs $\delta_1 = 1$, $\delta_2 = \pm 1$, $\delta_3 = \pm 1, \ldots$ such that the following relations hold:

(*a*) The equations (4) and (6) with $t_i = \delta_i$ for $1 \leqq i \leqq 4$ and $t_i = 0$ for $i > 4$.

(*b*) The equations (4) and (6a) in the following three cases:

1) $t_1 = \delta_1,\ t_4 = -\delta_4,\ t_6 = \delta_6,\ t_7 = \delta_7$, all other $t_i = 0$;
2) $t_2 = \delta_2,\ t_4 = -\delta_4,\ t_5 = \delta_5,\ t_7 = \delta_7$, all other $t_i = 0$;
3) $t_3 = \delta_3,\ t_4 = -\delta_4,\ t_5 = \delta_5,\ t_6 = \delta_6$, all other $t_i = 0$;

(*c*) $\delta_5\chi_5(\mu) = \delta_2\chi_2(\mu) + \delta_3\chi_3(\mu)$, $\delta_6\chi_6(\mu) = \delta_1\chi_1(\mu) + \delta_3\chi_3(\mu)$, $\delta_7\chi_7(\mu) = \delta_1\chi_1(\mu) + \delta_2\chi_2(\mu)$.

An elementary, but somewhat messy, computation allows to deduce $\chi_2(\mu) = x_2$. Again, G cannot be simple.

A more detailed discussion of the decomposition numbers leads to the following refinement of Theorem 1:

THEOREM 2. *Let G be a finite group whose 2-Sylow group is a quaternion group (ordinary or generalized). If H is the unique maximal normal subgroup of odd order, then G/H has a center of order 2.*

An example is given by the extended icosahedral group of order 120.

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1 Theorem 1 in the case that P is the ordinary quaternion group was obtained by R. Brauer and communicated to M. Suzuki who then also gave a proof. Both authors independently obtained the extension to the case that P is a generalized quaternion group.

2 Cf. Brauer, R., and K. A. Fowler, *Annals of Mathematics*, **62**, 565–583 (1955), in particular, Theorem (2A) and equation (23).

3 For the results used here, cf. Brauer, R., *Annals of Mathematics*, **42**, 926–935 (1941); these PROCEEDINGS, **32**, 215–219 (1946).

GENERALIZED SPACES OF GENERAL RELATIVITY

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1. This paper deals with a space of four dimensions V_4 of coordinates x^i as i takes the values 1 to 4. The geometry of the space is based in part on a set of asymmetric coefficients of connection Γ^i_{jk} in coordinates x^i which are related to the coefficients $\Gamma'^{\delta}_{\alpha\beta}$ in the coordinates x'^{α} by the equations

$$x^i_{,\alpha\beta} + \Gamma^i_{jk} x^j_{,\alpha} x^k_{,\beta} = \Gamma'^{j}_{\alpha\beta} x^i_{,\gamma} \tag{1}$$

Throughout this paper a quantity followed by a comma and an index denotes the derivatives of the quantity with respect to x or x' with this index as the case may be. Also when there is a repeated upper and lower index, the one term stands for the sum of terms as this index takes the values 1 to 4. Thus in equations (1) the second term stands for the sum of terms as j and k take the values 1 to 4, and in the right-hand member of the equation as γ takes the values 1 to 4.

When we express the condition of integrability of equations (1) and make use of these equations in the reduction, we obtain[1]

$$\Gamma^i_{jkl} x^j_{,\alpha} x^k_{,\beta} x^l_{,\gamma} = \Gamma'^{\sigma}_{\alpha\beta\gamma} x^i_{,\sigma}$$

where

$$\Gamma^i_{jkl} = \Gamma^i_{jk,l} - \Gamma^i_{jl,k} + \Gamma^i_{hl}\Gamma^h_{jk} - \Gamma^i_{hk}\Gamma^h_{jl} \tag{2}$$

and similarly for $\Gamma'^{\sigma}_{\alpha\beta\gamma}$. Hence Γ^i_{jkl} and $\Gamma'^{\sigma}_{\alpha\beta\gamma}$ are components of a tensor.

When in equation (2) the functions of the form Γ^i_{jk} are replaced by the Christoffel